Lefschetz properties of balanced 3-polytopes

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In this talk, we study Lefschetz properties of Artinian reductions of Stanley-Reisner rings of balanced simplicial 3-polytopes. A (d-1)-dimensional simplicial complex is said to be balanced if its graph is d-colorable. If a simplicial complex is balanced, then its Stanley-Reisner ring has a special system of parameters induced by the coloring. We prove that the Artinian reduction of the Stanley-Reisner ring of a balanced simplicial 3-polytope with respect to this special system of parameters has the strong Lefschetz property if the characteristic of the base field is not two or three. Moreover, we characterize (2,1)-balanced simplicial polytopes, i.e., polytopes with exactly one red vertex and two blue vertices in each facet, such that an analogous property holds. In fact, we show that this is the case if and only if the induced graph on the blue vertices satisfies a Laman-type combinatorial condition.

This is a joint work with David Cook II, Satoshi Murai and Eran Nevo.